

Quantum states and quantum gates with trapped ions





innsbru

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Outline

- Quantum information processing with trapped ions
- Generating and detecting entangled quantum states
- Quantum processes with trapped ions
- Non-unitary operations
- Summary and Outlook



A string of trapped ions



Qubits with trapped ions

Storing quantum information requires *long-lived atomic states*:

Optical transitions on metastable states S ⇔ D transitions in alkaline earths: Ca⁺, Sr⁺, Ba⁺, (Yb⁺, Hg⁺)



Innsbruck ⁴⁰Ca⁺

Microwave transitions on hyperfine or Zeeman states alkaline earths: ⁹Be⁺, ²⁵Mg⁺, ⁴³Ca⁺, ⁸⁷Sr⁺, ¹³⁷Ba⁺, ¹¹⁵Cd⁺, ¹⁷¹Yb⁺



Boulder ⁹Be⁺; Michigan ¹¹¹Cd⁺; Innsbruck ⁴³Ca⁺, Oxford ⁴³Ca⁺;

Ion species

lons in use:



from QC group C. Monroe, Univ. of Michigan

Choosing the right ion species:

- strong cooling transition
- availability of suitable laser sources

• ...

Level scheme of Ca⁺



Spectroscopy of the
$$S_{1/2} \Leftrightarrow D_{5/2}$$
 transition

Zeeman structure in non-zero magnetic field:



+ vibrational degrees of freedom

2-level-system:



lon traps

Goal: To trap a charged particle in 3 dimensions



lon traps

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Paul trap: Alternating electric field

Alternating electric field

Alternating attractive and repulsive force on the ion,

Effective trapping potential.

lon stays trapped.



Oscillating saddle potential



Linear Paul traps

Radial electrodes: Apply radiofrequency (20 MHz) with few hundred volts amplitude. Generates quadrupole potential.



Apply DC voltage of 500 – 2000 V. Provides axial confinement.





1. Initialization in a pure quantum state



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- 2. Quantum state manipulation on $S_{1/2} D_{5/2}$ transition



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A typical experimental sequence



Cycle is repeated for 100 – 200 times in order to determine the final quantum state of the ion string.









S

Addressing of individual ions

0 ∟ -10

-8

-6





- inter ion distance: $\sim 4 \ \mu m$

-2

Ω

2

Deflector Voltage (V)

8

10

- addressing waist: ~ 2 µm
- < 0.1% intensity on neighbouring ions

Letting the qubits interact

The common motion acts as the quantum bus. 0 0 0 0 0 0 0 0



50 µm

Letting the qubits interact

VOLUME 74, NUMBER 20

PHYSICAL REVIEW LETTERS

15 May 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universiät Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria (Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

PACS numbers: 89.80.+h, 03.65.Bz, 12.20.Fv, 32.80.Pj

...allows the realization of a **universal** quantum computer !

other gate proposals include:

- Cirac & Zoller
- Mølmer & Sørensen, Milburn
- Jonathan & Plenio & Knight
- Geometric phases
- Leibfried & Wineland

$$\begin{split} |D\rangle|D\rangle &\to |D\rangle|D\rangle \\ |D\rangle|S\rangle &\to |D\rangle|S\rangle \\ |S\rangle|D\rangle &\to |D\rangle|S\rangle \\ |S\rangle|S\rangle &\to |S\rangle|D\rangle \end{split}$$

control target

Common mode excitations

Center of mass mode position breathing mode 3v

Common mode excitations



A string of trapped ions coupled to the motion



A string of trapped ions coupled to the motion



Excitation spectrum of single ion in linear trap



Coherent state manipulation



carrier and sideband Rabi oscillations with Rabi frequencies

 Ω , $\eta \Omega \sqrt{n+1}$



 $\eta = k x_0$ Lamb-Dicke parameter

And that is how it looks like in real life....

...the ion trap inside the vacuum chamber...



And that is how it looks like in real life....



...and the optical table (2003)...

...and now with magnetic shielding.

Generation of Bell states

Generation of Bell states







Generation of Bell states


Generation of Bell states







Generation of Bell states





Entangled ions using a geometric gate:

- ⁹Be+: Boulder, D. Wineland
- ⁴⁰Ca⁺: Oxford, A. Steane
- ¹¹¹Cd⁺: Ann Arbor, C. Monroe

Analysis of Bell states

 $|SD\rangle + |DS\rangle$

Fluorescence detection with CCD camera:

Coherent superposition or incoherent mixture ?

What is the relative phase of the superposition ?

→ Measurement of the density matrix:





Measuring a density matrix

A measurement yields the *z*-component of the Bloch vector

 \rightarrow Diagonal of the density matrix

$$\rho = \left(\begin{array}{cc} \mathbf{P}_{\mathbf{S}} & C - iD \\ C + iD & \mathbf{P}_{\mathbf{D}} \end{array}\right)$$



Measuring a density matrix

A measurement yields the *z*-component of the Bloch vector

 \rightarrow Diagonal of the density matrix

$$\rho = \left(\begin{array}{cc} P_S & C - iD \\ C + iD & P_D \end{array}\right)$$

Rotation around the *x*- or the *y*-axis prior to the measurement yields the phase information of the qubit.





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 \rightarrow coherences of the density matrix





Tomography of Bell states



C. Roos, et al, Phys. Rev. Lett. 92, 220402 (2004).

Generation of W-states



W - states $|\Psi\rangle = \frac{1}{\sqrt{3}} \left(|SDD\rangle + |DSD\rangle + |DDS\rangle \right)$



experimental result

theoretical expectation

C. Roos et al., Science 304, 1478 (2004)

W-states











 $\frac{1}{\sqrt{N}}(|SS\dots SD\rangle + |SS\dots SDS\rangle + |DS\dots SS\rangle)$

Häffner et al., Nature 438, 643 (2005)

W-states



Generation of high-fidelity GHZ states

Generation of high-fidelity GHZ states



$$|\mathrm{GHZ}_N
angle = rac{1}{\sqrt{2}} \left(|00\dots0
angle + |11\dots1
angle
ight)$$

Why interesting?

- highly non-classical: test fundamental physics (locality, Bell inequalities)
- applications: quantum key distribution, quantum communication
- quantum metrology (enhanced phase sensitivity)

High-fidelity gate operation



Deterministic Bell states with the Mølmer-Sørensen gate



Measuring the entanglement

measure the parity [Sackett et al., Nature 404, 256 (2000)]



$$\prod = \langle \prod_{j=1}^{N} \sigma_j^z \rangle$$

Parity Π oscillates with N ϕ



Bell / GHZ – Fidelity:

$$F = \frac{1}{2}(P_{SS...S} + P_{DD...D} + C)$$

Witness:

$$W = 1 - 2 \cdot C$$

C: amplitude of the parity oscillations

Mølmer-Sørensen gate: parity oscillations

Parity oscillation contrast: $|\langle SS|\rho_{\Psi}|DD\rangle|$



Bell state: $\Psi = |SS\rangle + i|DD\rangle$

C = 0.990(1) 29,400 measurements $p_{SS}+p_{DD} = 0.9965(4)$ 13,000 measurements Bell state fidelity F=99.3(1)%



N - qubit GHZ state generation with global MS gates



N - qubit GHZ state generation with global MS gates



State detection using a photomultiplier

Probability (a.u.)



See: Thomas Monz, PhD thesis (2011).

Remember: We want to know the populations and the coherences.

Bell / GHZ – Fidelity:

$$F = \frac{1}{2}(P_{SS...S} + P_{DD...D} + C)$$

Witness:

 $W = 1 - 2 \cdot C$

C: amplitude of the parity oscillations



Bayesian analysis of the GHZ coherence



See: Thomas Monz, PhD thesis (2011).

Decay of N-qubit GHZ states



T. Monz et al., PRL 106, 130506 (2011).

Scaling of entanglement decay



Dephasing model

Master equation - dephasing (e.g. Huelga et al., PRL 79, 3865 (1997)]

Uncorrelated dephasing of qubits

$$Contrast \propto e^{-N\gamma t}$$

But we observe: correlated dephasing

$$Contrast \propto e^{-N^2 \gamma t}$$

Does that make sense?

Sources of dephasing

Laser vs. atomic transition frequency fluctuations by

- a. Laser noise
- b. Magnetic field fluctuations

Effect all ions equally! \rightarrow Correlated!



Solution: Use decohence free subspace against dephasing.

Quantum process tomography of the quantum Toffoli operation

Toffoli gate: controlled-controlled NOT

Toffoli gate (Tommaso Toffoli, 1980):

..... is a universal reversible logic gate, i.e. any reversible circuit can be constructed from Toffoli gates.

also known as the controlled-controlled-NOT or CCNOT-gate operation



useful, e.g. for error correction

...the basic implementation idea

- 1. Map the combined logical information onto the motion
- 2. Perform a standard Cirac-Zoller CNOT gate
- 3. Unmap the motional information



Toffoli gate: pulse sequence

use 2-phonon excitation

T. Monz, K. Kim et al., PRL 103, 200503 (2009).



Toffoli gate: experimental truth table



Quantum process tomography



$$\rho_{\rm out} = \sum \chi_{ij} E_i \rho_{\rm in} E_j^{\dagger}$$

$$E_i = A_i \otimes A_j$$
$$A_i \in \{I, \sigma_x, \sigma_y, \sigma_z\}$$



characterizes gate operation completely

Toffoli gate: process tomography




Toolbox for generating

high-fidelity operations

and

dissipation





Entangling gate: Mølmer-Sørensen



Realizes a two-body Hamiltonian where every ion interacts with every other, e.g., for 3 ions

$$S_x^2 = \sigma_x^{(0)} \sigma_x^{(1)} + \sigma_x^{(1)} \sigma_x^{(2)} + \sigma_x^{(0)} \sigma_x^{(2)}$$

Together a toolbox for arbitrary unitary operations!

Basic set of operations:

collective spin flips

individual light shift gates

Mølmer-Sørensen gate

$$egin{aligned} S_x, S_y \ \sigma_z^{(0)}, \sigma_z^{(1)}, \sigma_z^{(2)} \ S_x^2 \end{aligned}$$



Arbitrary unitary operations can be achieved !

Qubit reset by optical pumping = dissipation



Note: Scattered photons carry information about previous ionic quantum state. When entangled, tracing over photonic bath leads to decoherence.

Qubit reset by optical pumping = dissipation



Can be used for:

- qubit reset
- controlled decoherence (after tracing over bath)
- removing entropy
- optical pumping into entangled states
- simulating general open-system dynamics

Repeated quantum error correction

Quantum error correction algorithm

Qubit reset = removes error syndrome



Repetition code with quantum feedback

Encoding

$$\begin{split} |\Psi\rangle &= \alpha |0\rangle + \beta |1\rangle \quad \square\rangle \quad |1\rangle & \bigoplus \quad |1\rangle & \bigoplus \quad \alpha |000\rangle + \beta |111\rangle \\ &|1\rangle & \bigoplus \quad |1\rangle & \bigoplus \quad |1\rangle & \oplus \quad |1\rangle$$

Measurement-free decoding



Creating the pulse sequence – part I



Adapted quantum repetition code

Algorithm corrects for phase flip errors



Optimizing the algorithm

Modified gradient ascent pulse engineering (GRAPE) algorithm

Pulse sequence:
$$U = \prod_m e^{-i \Theta_m H_m}$$

Performance function Φ that has maximum for exact algorithm.

Optimize pulse length along gradient:

$$\Theta_m = \Theta_m + \frac{\partial \Phi}{\partial \Theta_m}$$

Add random pulses Hm from time to time.

V. Nebendahl et al., Phys. Rev. A 79, 012312 (2009).

Creating the pulse sequence – part II





Optimization procedure



Multiple repetitions of the QEC algorithm





Quantum error correction of phase noise



Thanks to...

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ERC, Marie Cure, aqute Industrie Tirol



🚀 IQI GmbH



