Detecting Quantum Light
Part II

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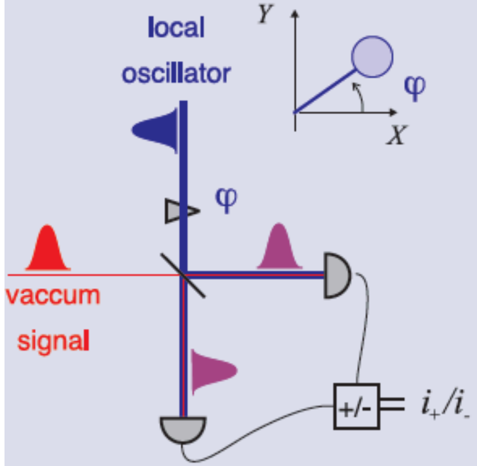
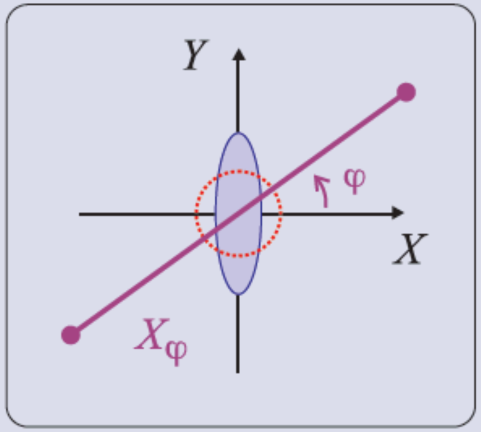
Bad Honnef

- Detector tomography
- Measuring correlated photon statistics
- Direct probing of the Wigner function

Detection scheme





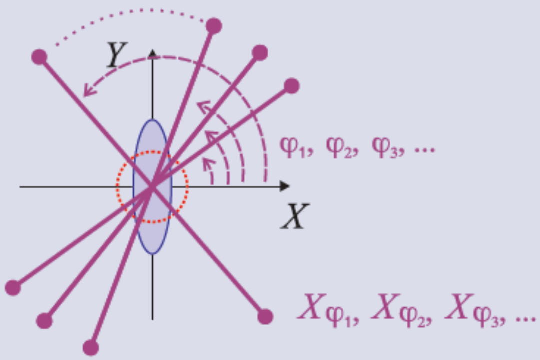
semiconductor photo-diodes

quadrature measurement:

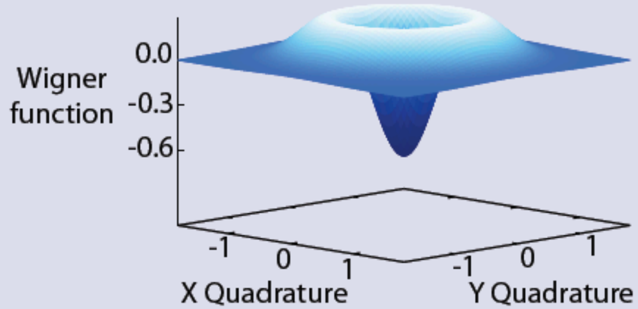
$$\hat{X}_\varphi = \hat{a}e^{-i\varphi} + \hat{a}^\dagger e^{i\varphi}$$

$$\langle \hat{N}_d - \hat{N}_c \rangle \propto \beta^* \, \hat{a} + \beta \, \hat{a}^\dagger =$$

$$|\beta| \, \langle \hat{X}^a_\varphi \rangle$$



$\Rightarrow$



Quantum measurement:

$$p_{n,\hat{\rho}} = \mathrm{tr}\,[\hat{\rho}\,\hat{\pi}_n]$$

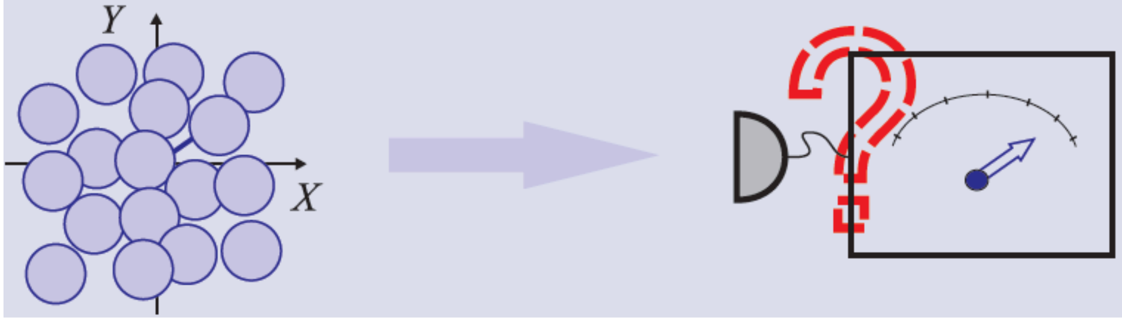
$\hat{\pi}_n$ POVM: describes detector response, typically assumed to be known

quantum state tomography: $\hat{\pi}_1, \hat{\pi}_2, \hat{\pi}_3, ...$ $\Rightarrow$ state reconstruction

Concept

interchange role of states and POVM

detector tomography: $\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3, ...$ $\Rightarrow$ measurement operator reconstruction



$\Rightarrow$ POVM: $\hat{\pi}_n$

A. Luis, L.L. Sanchez-Soto, Phys. Rev. Lett 83, 3573 (1999).

Detector without phase reference: (verified in experiment) $\hat{\pi}_k = \sum_j \theta_j^{(k)} |j\rangle\langle j|$

$$p_k = \mathrm{tr}\left(\hat{\pi}_k \hat{\rho}\right) \quad \Leftrightarrow \quad \vec{p} = C\, L_\eta\, \vec{\rho}_{nn}$$

describes detector response = theoretical model for POVMs

rows of $C\, L_\eta$ define POVMs: $\left[C\, L_\eta\right]_{kj} = \theta_j^{(k)}$

Define matrix equation (analogous to TMD analysis)

$$\mathrm{P} = \mathrm{F}\,\Pi, \quad \text{with } \mathrm{P}_{\mathrm{D}\times(\mathrm{M}+1)}, \text{ and } \mathrm{F}_{\mathrm{D}\times\mathrm{N}}\ , \Pi_{N\times(M+1)}$$

$D:$ $\alpha_1, \ldots, \alpha_D$ probe states

$k = 0, \ldots, M:$ possible detector outcomes, M port TMD

$n = 0, \ldots, N:$ relevant photon number contributions (higher photon numbers do not change POVMs)

F : number statistics of probe states

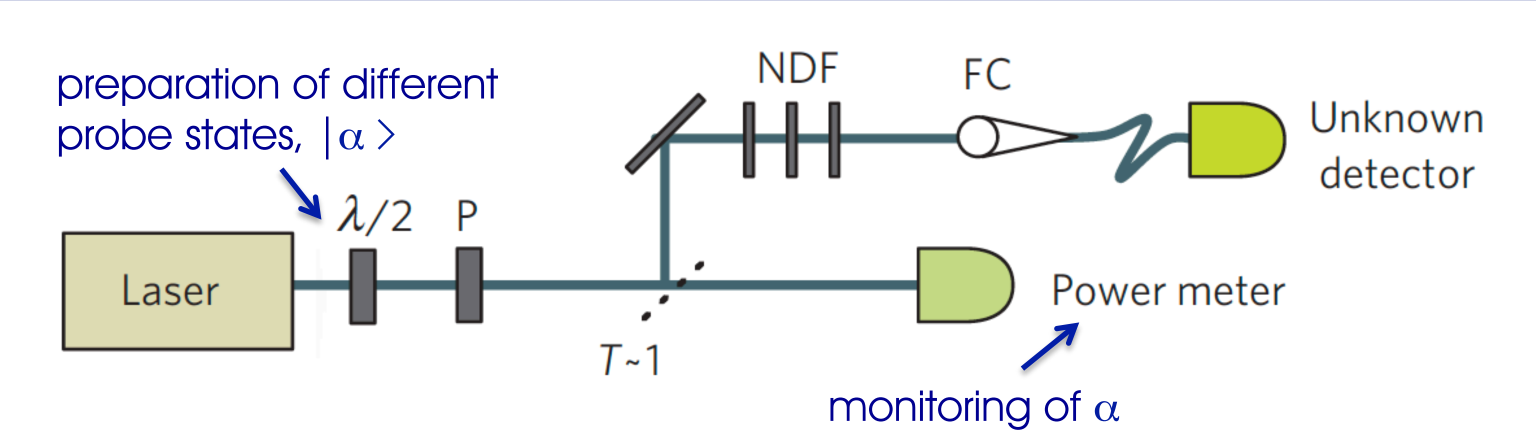
P : probabilities of k "clicks"

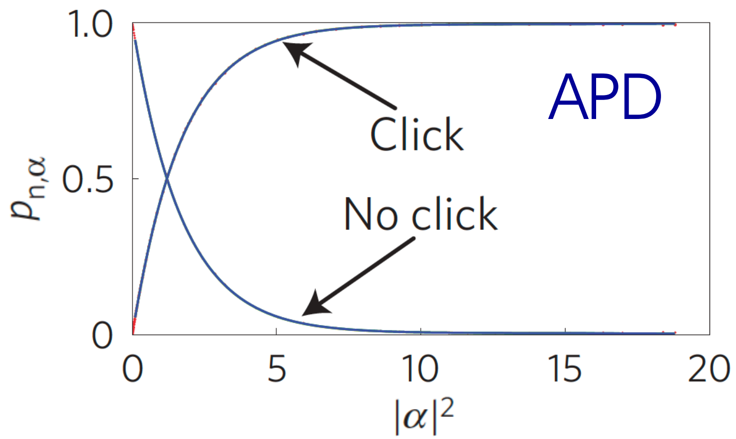
Π : coefficients of POVMs

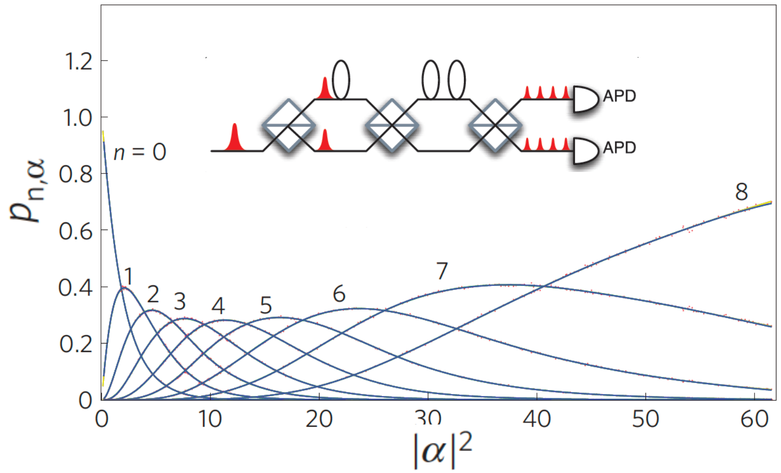
Optimization problem for Π:

$$\min\left\{\left\|\mathrm{P} - \mathrm{F}\,\Pi\right\|\right\} \text{ under constraints } \hat{\pi}_k \geq 0, \quad \sum_{k=0}^{M} \hat{\pi}_k = 1$$

Experimental setup





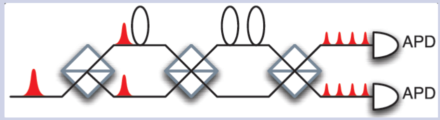


J.S. Lundeen, A. Feito, H. Coldenstrodt-Ronge, K.L. Pregnell, Ch. S., T.C. Ralph, J. Eisert, M.B. Plenio, I.A.Walmsley, Nature Physics, 5, 27 (2009).

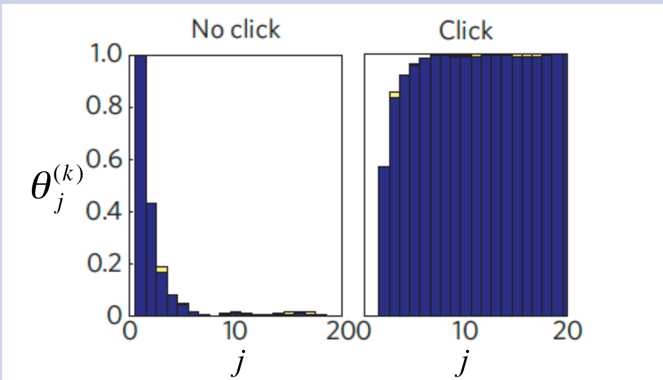
TMD tomography

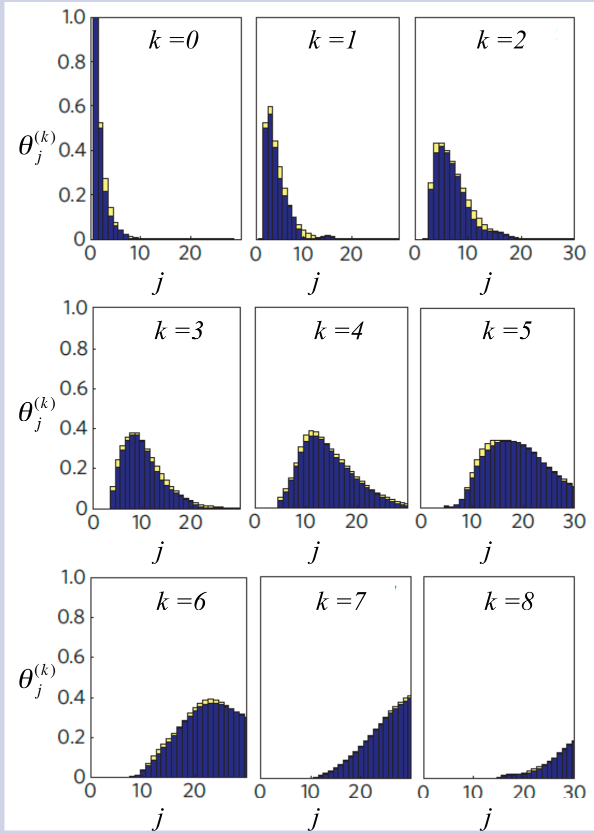
POVMs

TMD



$$\hat{\pi}_k = \sum_j \theta_j^{(k)} |j\rangle\langle j|$$





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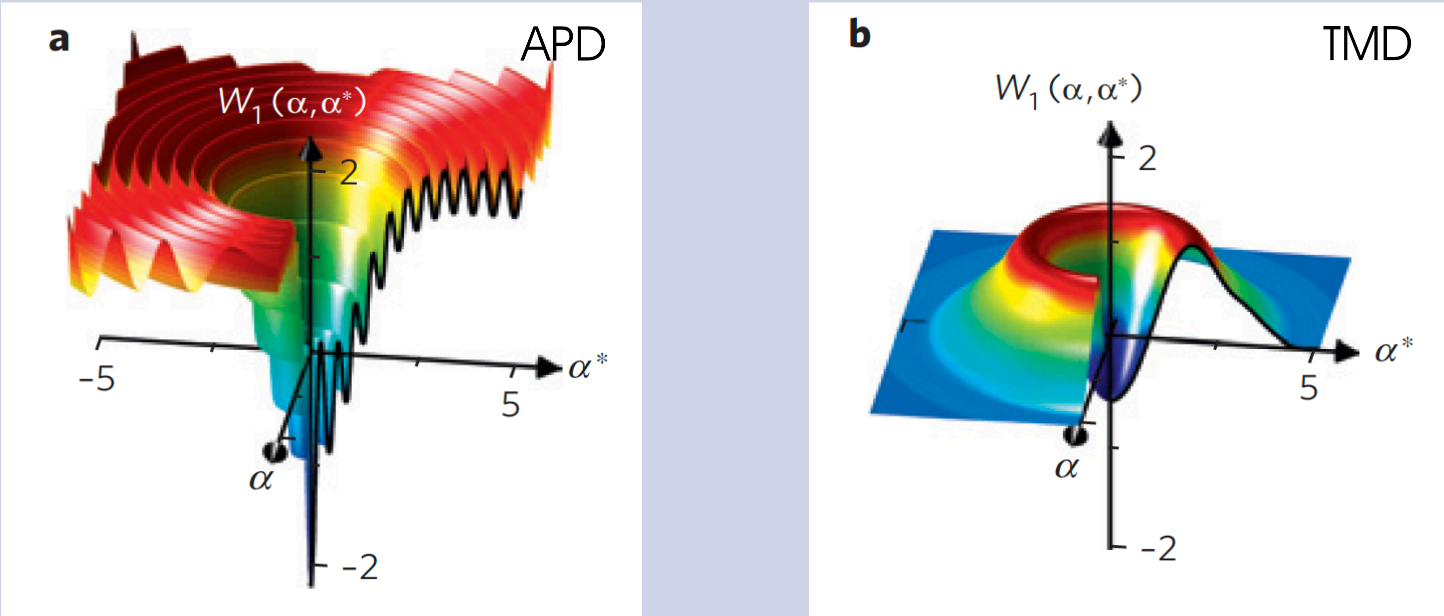
POVM for single „click“

$$p_{n,\psi} = \int W_n W_\psi \, d\alpha \, d\alpha^*$$



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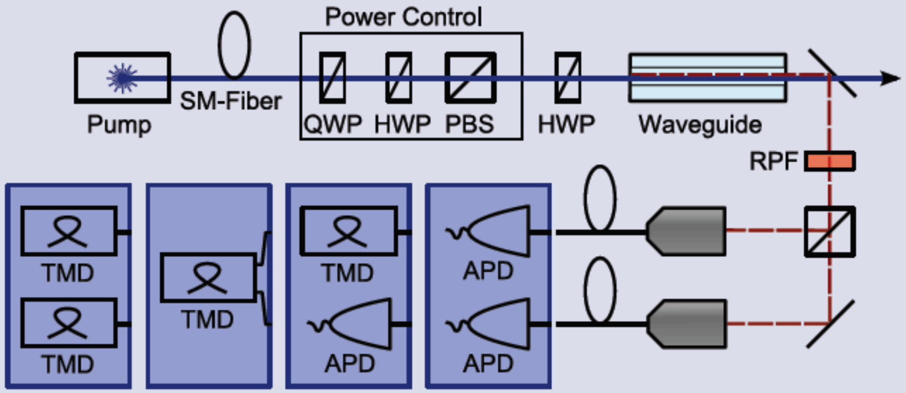
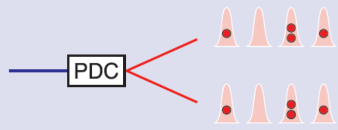
Parametric downconversion



correlated photon number

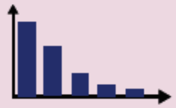


$$|\Psi\rangle = \sum_{n=0}^{\infty} c_n \, |n\rangle \, |n\rangle$$

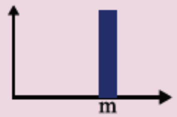
$$p_n = L^{-1}(\eta) \cdot C^{-1} p_k$$

Photon statistics / correlations

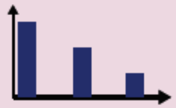
MARGINAL: $p(\text{signal} = n) = p_n \quad p(\text{idler} = n) = p_n$



CONDITIONED: $p(\text{signal} = n | \text{idler} = m) = \delta_{m,n}$

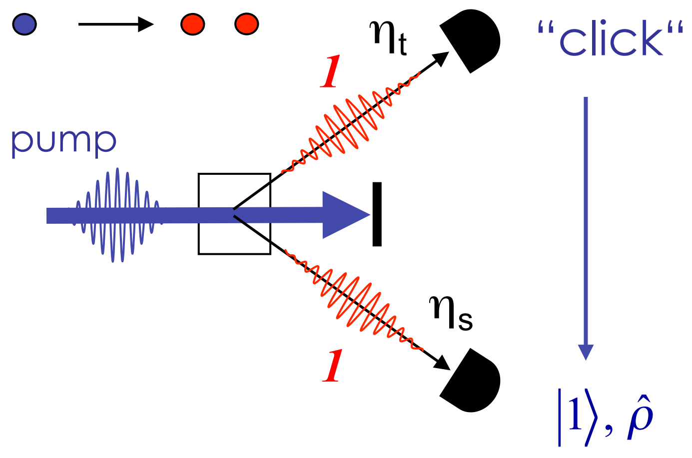


SUM: $p(\text{sum} = n \text{ signal} + m \text{ idler}) = \delta_{m,n} \, p_n$



Self-referencing detector

PDC at low gain



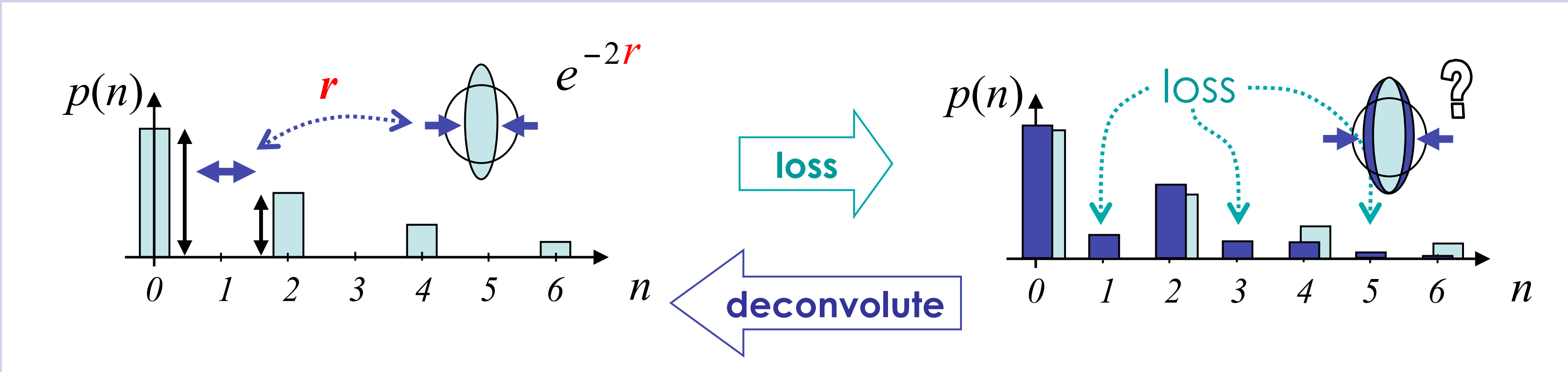
A priori information:
photons are produced in pairs

$$R_s = \eta_s \cdot R \ , \quad R_t = \eta_t \cdot R$$

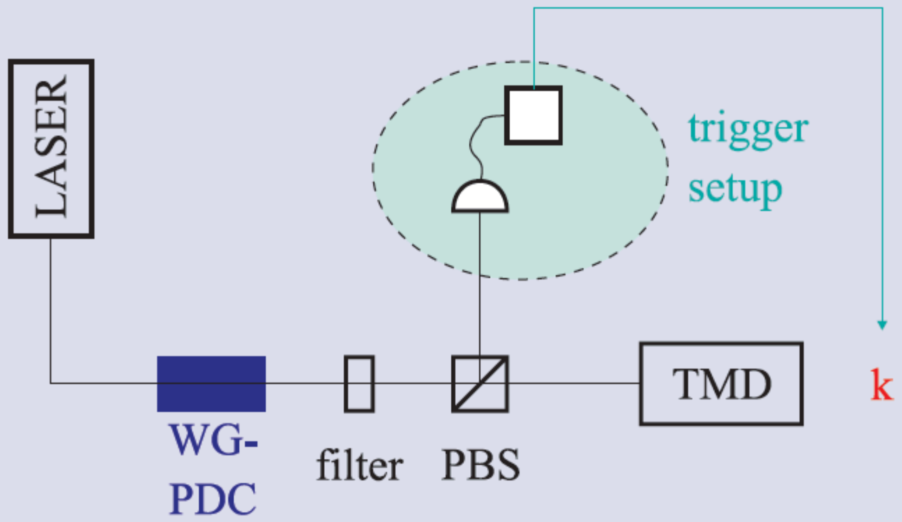
$$R_{coin} = \eta_s \ \eta_t \cdot R$$

$$\Rightarrow \quad \eta_s = \frac{R_{coin}}{R_t}$$

Loss-tolerant characterization of photon number statistics (use a priori information)



Conditional preparation of single-photon states



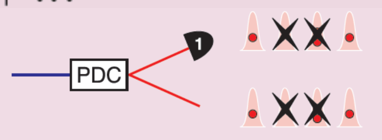
Correlated photon numbers

$$|\Psi\rangle = \sum_{n=0}^{\infty} c_n |n\rangle |n\rangle$$

$$p_n = L^{-1}(\eta) \cdot C^{-1} p_k$$

Measurement of mode 1:

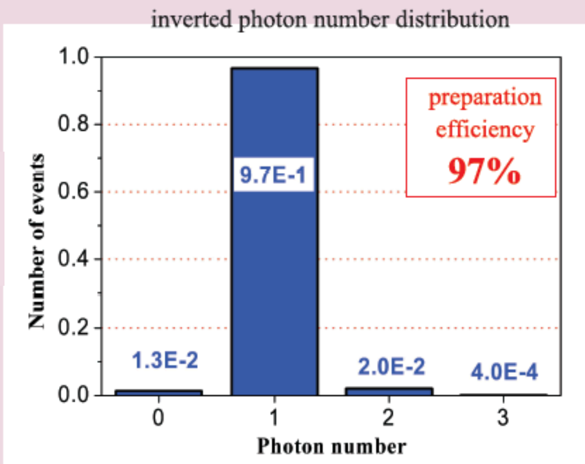
$$\hat{M} = |1\rangle\langle 1| + |2\rangle\langle 2| + \ldots$$



State in mode 2:

$$\mathrm{tr}(\hat{M}\,|\Psi\rangle\langle\Psi|) = N \sum_{n=1}^{\infty} c_n |n\rangle$$

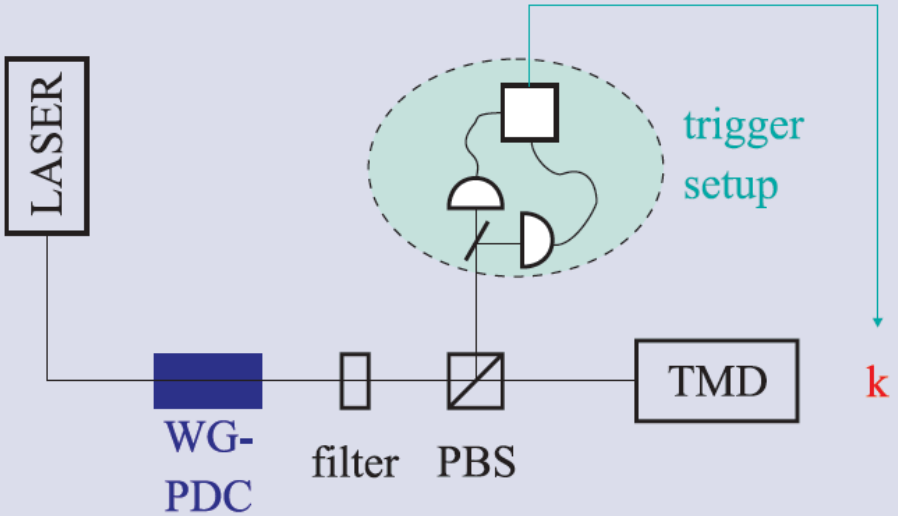
detection propability: 37,5 %



D. Achilles, Ch. Silberhorn, and I.A. Walmsley Phys. Rev. Lett. 97, 043602 (2006)

PDC photon number statistics

Conditional preparation of single-photon states



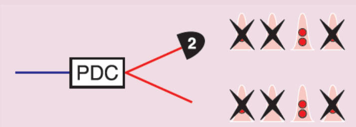
Correlated photon numbers

$$|\Psi\rangle = \sum_{n=0}^{\infty} c_n |n\rangle |n\rangle$$

$$p_n = L^{-1}(\eta) \cdot C^{-1} p_k$$

Measurement of mode 1:

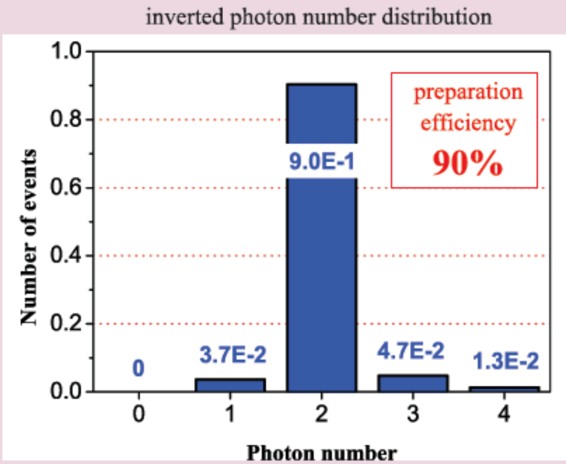
$$\hat{M} = |2\rangle\langle 2| + |3\rangle\langle 3| + \ldots$$



State in mode 2:

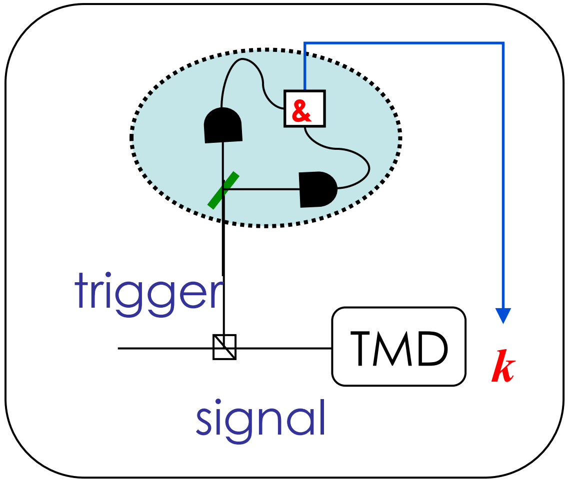
$$\mathrm{tr}(\hat{M}|\Psi\rangle\langle\Psi|) = N \sum_{n=2}^{\infty} c_n |n\rangle$$

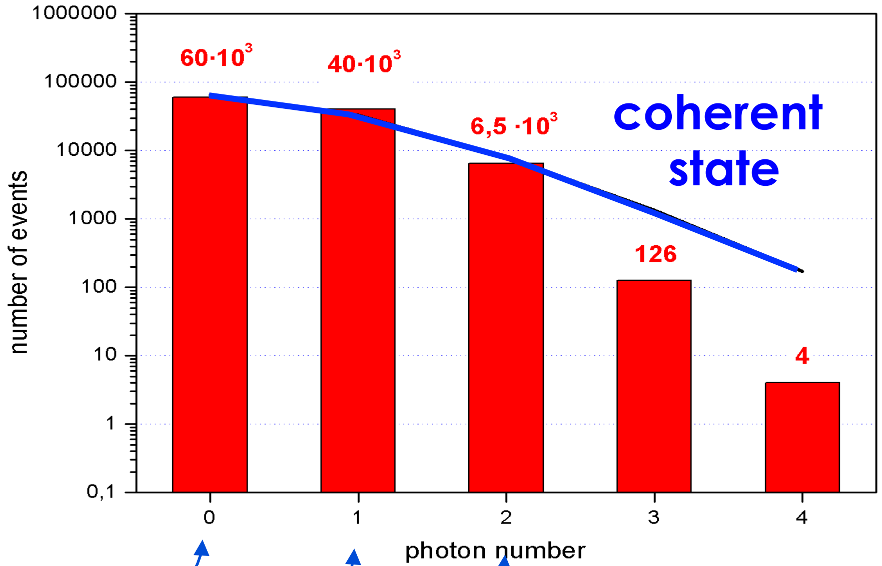
detection propability: 31,5 %



D. Achilles, Ch. Silberhorn, and I.A. Walmsley Phys. Rev. Lett. 97, 043602 (2006)

Detector calibration: estimation of losses





η_s : losses in signal arm

$p(k\,|\,t_c)$: count probability conditioned on coincidence trigger

$$p(k=0\,|\,t_c) = (1-\eta_s)^2 \Longrightarrow 31{,}5 \pm 0.1\ \%$$

$$p(k=1\,|\,t_c) = 2\eta_s(1-\eta_s) \Longrightarrow 31{,}0 \pm 0.2\ \%$$

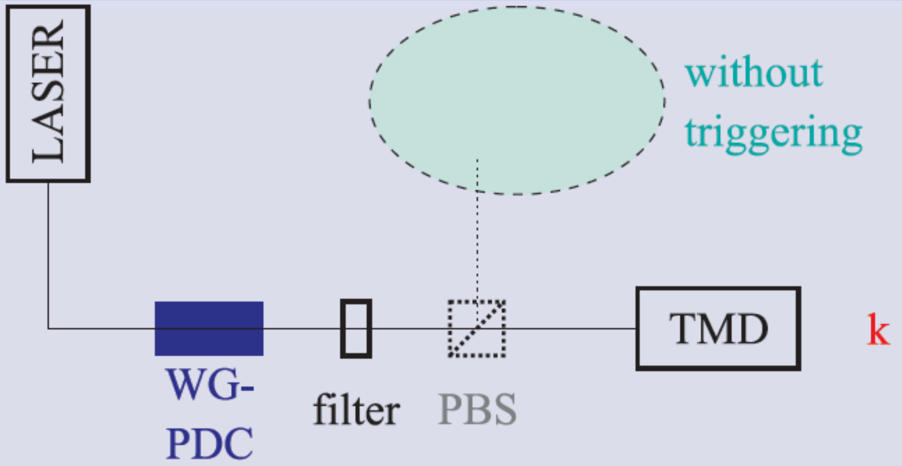
$$p(k=2\,|\,t_c) = \eta_s^{\,2} \Longrightarrow 32{,}1 \pm 0.2\ \%$$

Consistency Test:
very sensitive to higher photon numbers

D. Achilles, Ch. Silberhorn, and I.A. Walmsley Phys. Rev. Lett. 97, 043602 (2006)

PDC photon number statistics

PDC at higher gains



Correlated photon numbers

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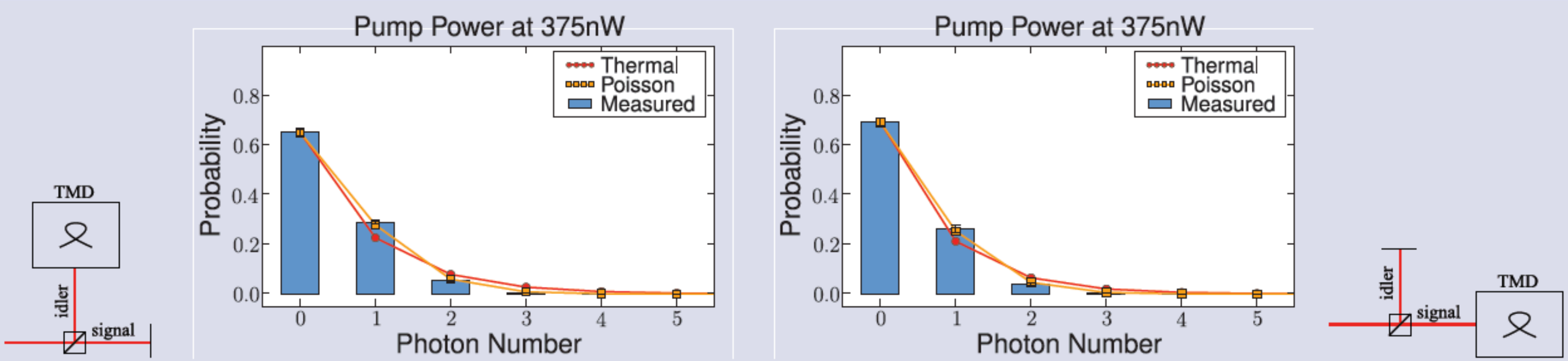
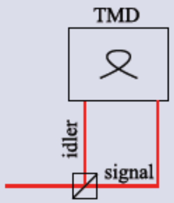
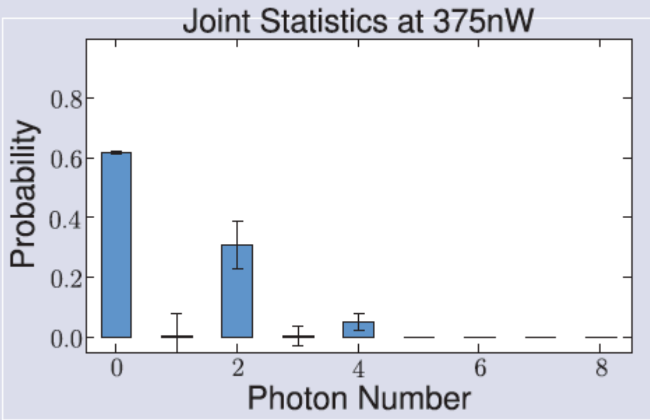
$$p_n = L^{-1}(\eta) \cdot C^{-1} p_k$$

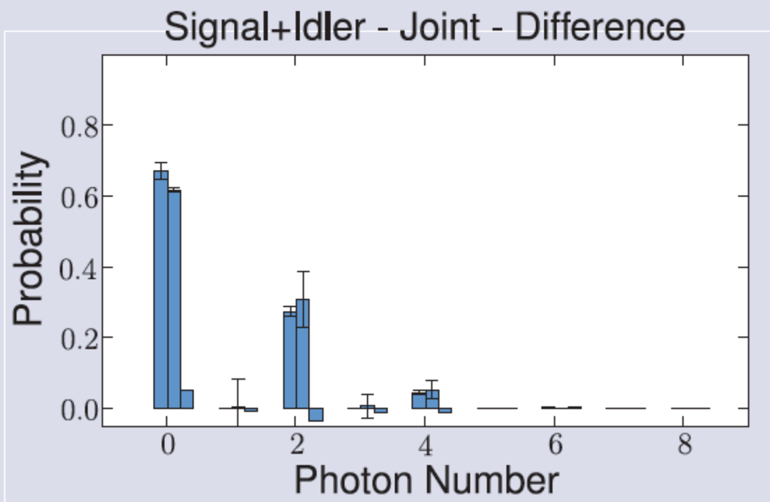


M. Avenhaus, H. B. Coldenstrodt-Ronge, K. Laiho, I. A. Walmsley, Ch. S.,
Phys. Rev. Lett. 101, 053601 (2008)

PDC at higher gains



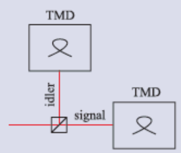




Measurements of correlations, shot by shot:

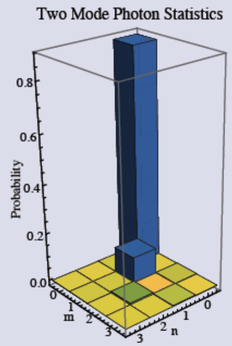
strict photon number correlations

joint statistics



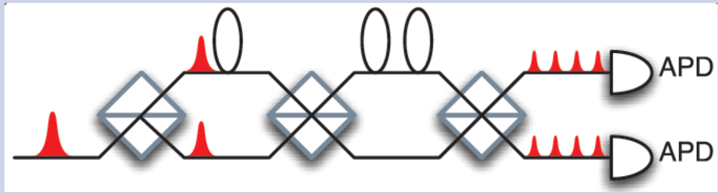
$$\frac{\Delta^2(n_s - n_i)}{\langle n_s\rangle\langle n_i\rangle} = \text{-23.9 dB}$$

(raw data: -0.2 dB)



M. Avenhaus, H. B. Coldenstrodt-Ronge, K. Laiho, I. A. Walmsley, Ch. S., Phys. Rev. Lett. 101, 053601 (2008)

Shot by shot TMD measurements



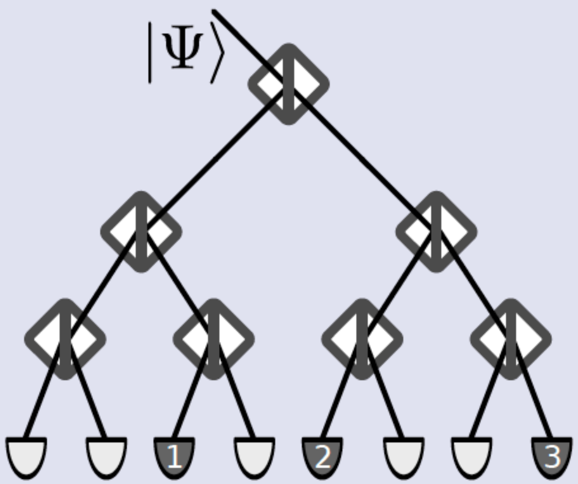
MATRIX STATISTICS

Cond.Loopy (CLoopySM.txt)

284695919	13216678	273235	3326	21	0	0	0	0
1473677	270166	9407	163	1	0	0	0	0
3807	1118	118	2	0	0	0	0	0
6	5	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0

correlation matrix

Spatial implementation



Measurement

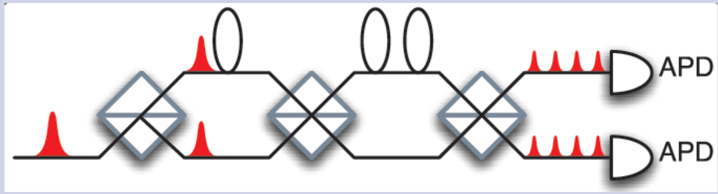
1. choose n Detectors
2. coincidences $\langle \hat{\mathbf{a}}_1^\dagger \hat{\mathbf{a}}_1 \cdots \hat{\mathbf{a}}_n^\dagger \hat{\mathbf{a}}_n \rangle$
3. intensity $\langle \hat{\mathbf{a}}_1^\dagger \hat{\mathbf{a}}_1 \rangle \cdots \langle \hat{\mathbf{a}}_n^\dagger \hat{\mathbf{a}}_n \rangle$
4. input mode: $\hat{\mathbf{a}}_j = \sqrt{\eta_j}\, \hat{\mathbf{a}} + \sqrt{1-\eta_j}\, \hat{\mathbf{v}}$

$$\Longrightarrow \frac{\eta_1 \cdots \eta_n \langle \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}}^\dagger \ldots \hat{\mathbf{a}} \hat{\mathbf{a}} \rangle}{\eta_1 \langle \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}} \rangle \cdots \eta_n \langle \hat{\mathbf{a}}^\dagger \hat{\mathbf{a}} \rangle} = g^{(n)}$$

loss independent!

M. Avenhaus, K. Laiho, M. V. Chekhova, and C. Silberhorn,
Phys. Rev. Lett. 104, 063602 (2010)

Shot by shot TMD measurements



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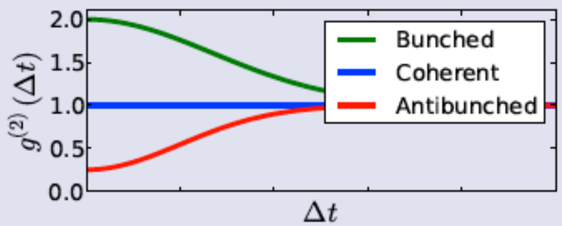
3807	1118	118	2	0	0	0	0	0
6	5	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0

correlation matrix

Quantum optics: Glauber correlation functions

$$g^{(n)} = \frac{\langle \hat{\mathbf{a}}^\dagger(t_1)\cdots\hat{\mathbf{a}}^\dagger(t_n)\hat{\mathbf{a}}(t_n)\cdots\hat{\mathbf{a}}(t_1)\rangle}{\langle \hat{\mathbf{a}}^\dagger(t_1)\hat{\mathbf{a}}(t_1)\rangle\cdots\langle \hat{\mathbf{a}}^\dagger(t_n)\hat{\mathbf{a}}(t_n)\rangle}$$

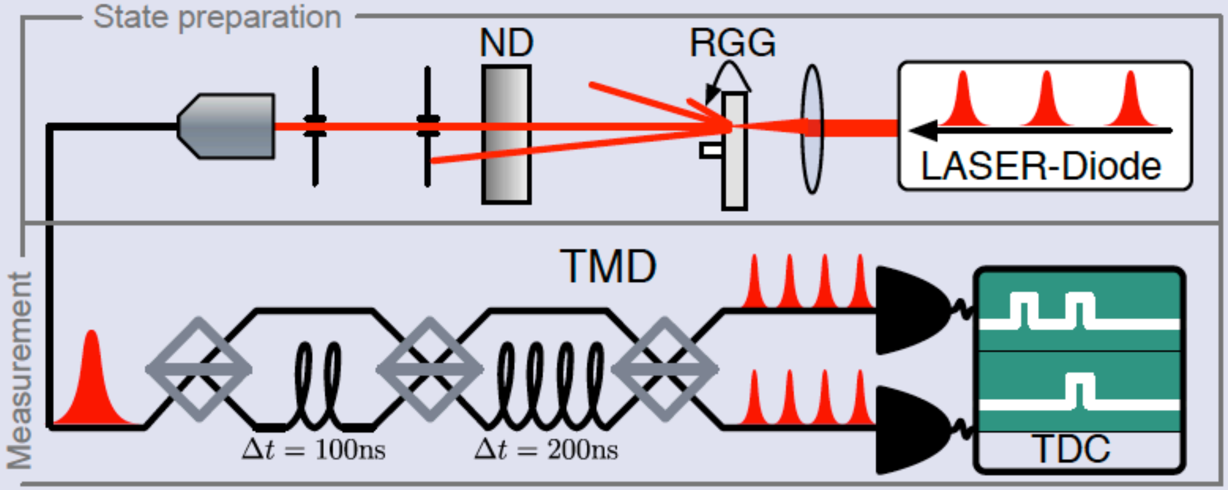
$$g^{(2)}(0)\begin{cases} >1 & \text{photon bunching} \\ =1 & \text{coherent light} \\ <1 & \text{photon antibunching} \end{cases}$$



M. Avenhaus, K. Laiho, M. V. Chekhova, and C. Silberhorn,
Phys. Rev. Lett. 104, 063602 (2010)

Measuring correlations functions

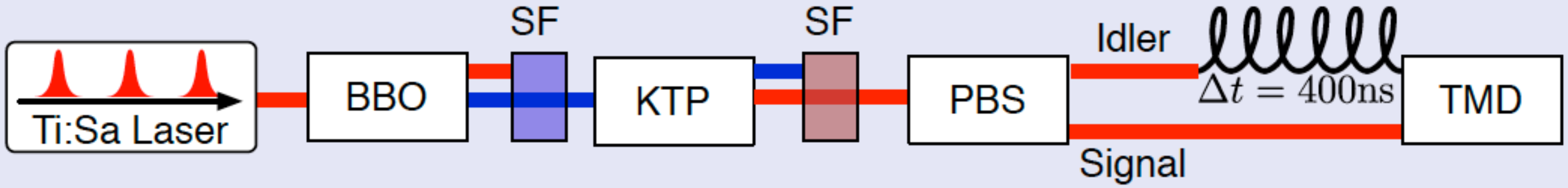
Experimental Setup



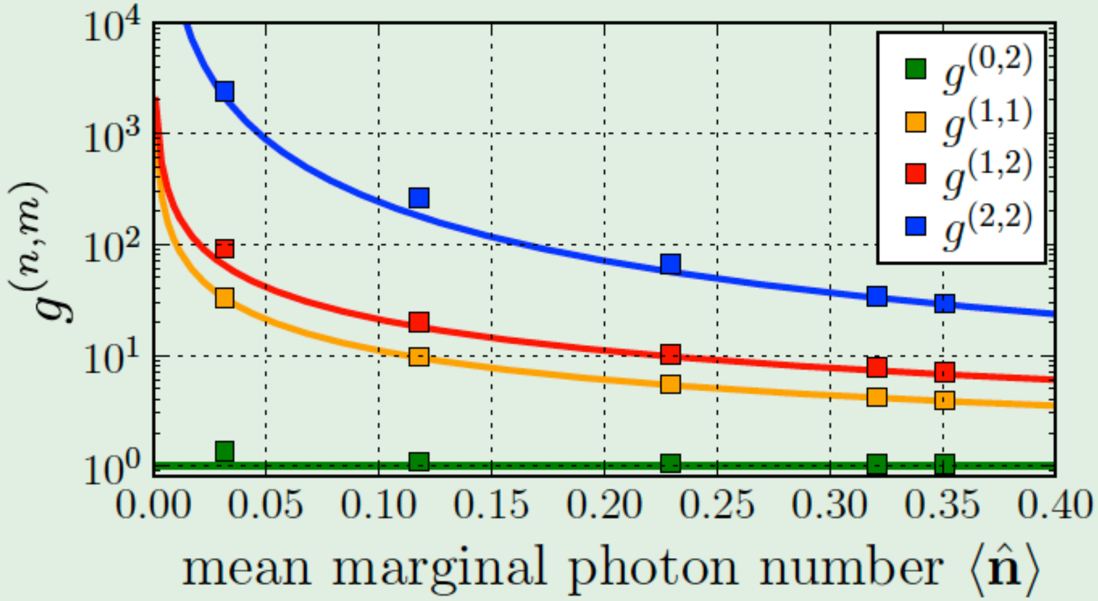
Measurement data[3]

	$g^{(2)}$	$g^{(3)}$	$g^{(4)}$	$g^{(5)}$	$g^{(6)}$	$g^{(7)}$	$g^{(8)}$	
Coherent	1.000	1.000	1.000	1.000	1.000	1.000	1.000	T
	1.000	1.001	1.002	1.002	1.003	1.005	0.987	E
Chaotic	2.00	6.00	24.00	120.00	720.00	5040	40320	T
	2.03	6.28	25.23	120.23	651.54	—	—	E

M. Avenhaus, K. Laiho, M. V. Chekhova, and C. Silberhorn,
Phys. Rev. Lett. 104, 063602 (2010)



Nonclassicality criteria[4] : $g^{(1,2)} > \sqrt{g^{(2,2)} g^{(0,2)}}$



$\Longrightarrow$ violates classicality by $g^{(1,2)} = 1.19...1.60 \sqrt{g^{(2,2)} g^{(0,2)}}$

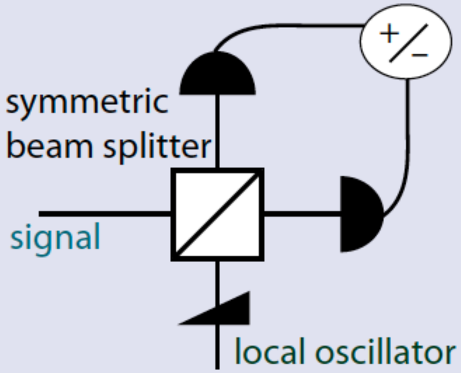
M. Avenhaus, K. Laiho, M. V. Chekhova, Ch. S:, Phys. Rev. Lett. 104, 063602 (2010)

[4] W. Vogel, Phys. Rev. Lett. 100, 013605 (2008)

- Detector tomography
- Measuring correlated photon statistics
- Direct probing of the Wigner function

Measuring Wigner functions

Balanced homodyning

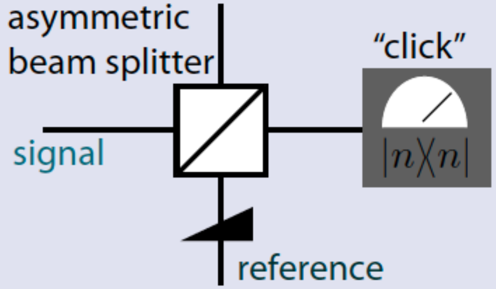


- ✔ Strong signal amplification
- ✔ Tomographic reconstruction
- ✘ Intrinsic filtering operation

$$\Delta^2 I = |\alpha|^2 \Delta^2 X^\theta$$

K. Vogel and H. Risken, PRA 40, 2847(1989)

Direct probing



- ✔ Weak displacement
- ✔ Direct reconstruction via photon number parity,

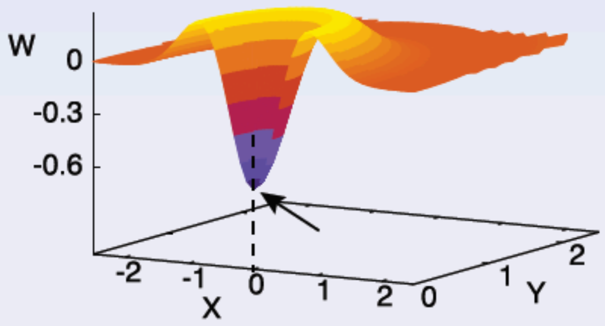
$$W(\alpha) = \frac{2}{\pi} Tr[\hat{D}^\dagger(\alpha)\rho\hat{D}(\alpha)\hat{\Pi}]$$

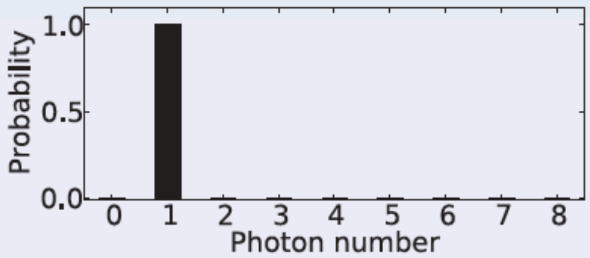
- ✔ Sensitive to all modes

K. Banaszek, K. Wodkiewicz, PRL, 76, 4344(1996);
S. Wallentowitz, W. Vogel, PRA, 53, 4528(1996)

Direct probing of Wigner functions

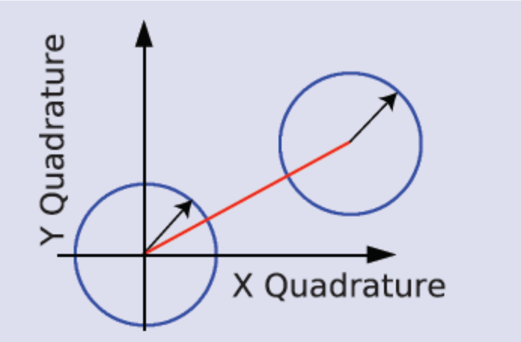
$\hat{D}(0)\,|1\rangle$:





Wigner function

$$
\begin{aligned}
W(0) &= \frac{2}{\pi}\left\langle \hat{\Pi} \right\rangle \\
&= \frac{2}{\pi}\sum_n (-1)^n P_n
\end{aligned}
$$

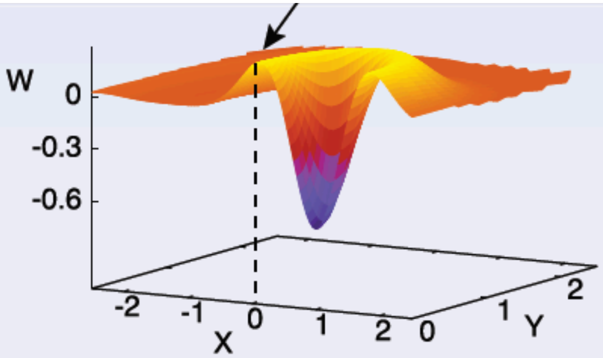


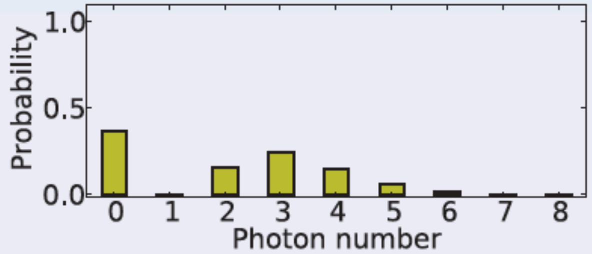
S. Wallentowitz, W. Vogel, PRA 53 (4528), 1996

K. Banaszek, K. Wodkiewicz, PRL 76 (4344), 1996

Direct probing of Wigner functions

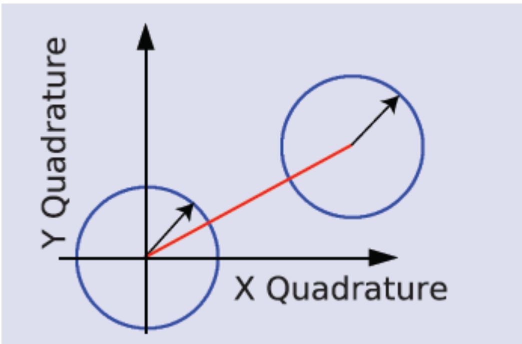
$\hat{D}(1)\,|1\rangle$:





Wigner function

$$
\begin{aligned}
W(-\alpha) &= \frac{2}{\pi}\left\langle \hat{\Pi} \right\rangle_{\alpha} \\
&= \frac{2}{\pi}\sum_{n}(-1)^{n}P_{n}(\alpha)
\end{aligned}
$$



S. Wallentowitz, W. Vogel, PRA 53 (4528), 1996

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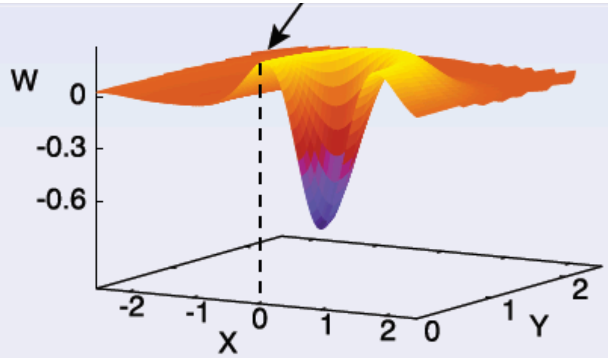

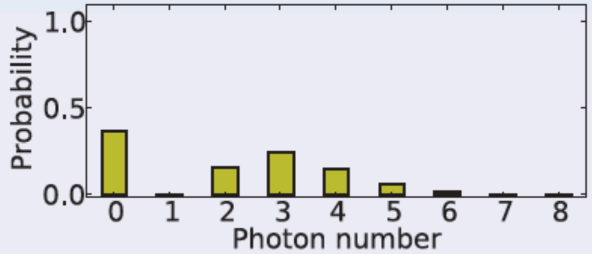

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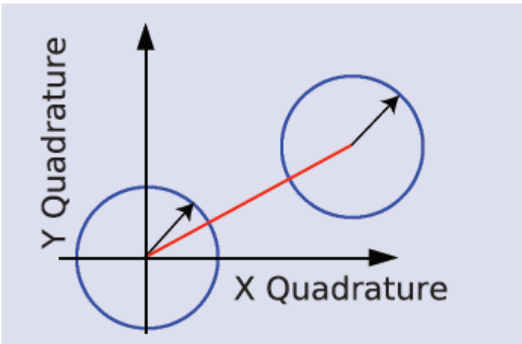
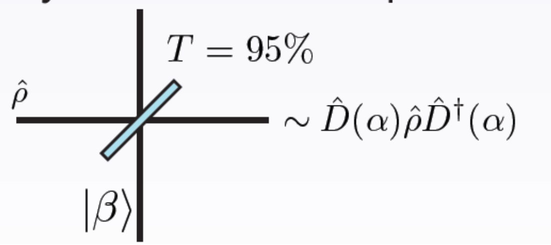


asymmetric beam splitter



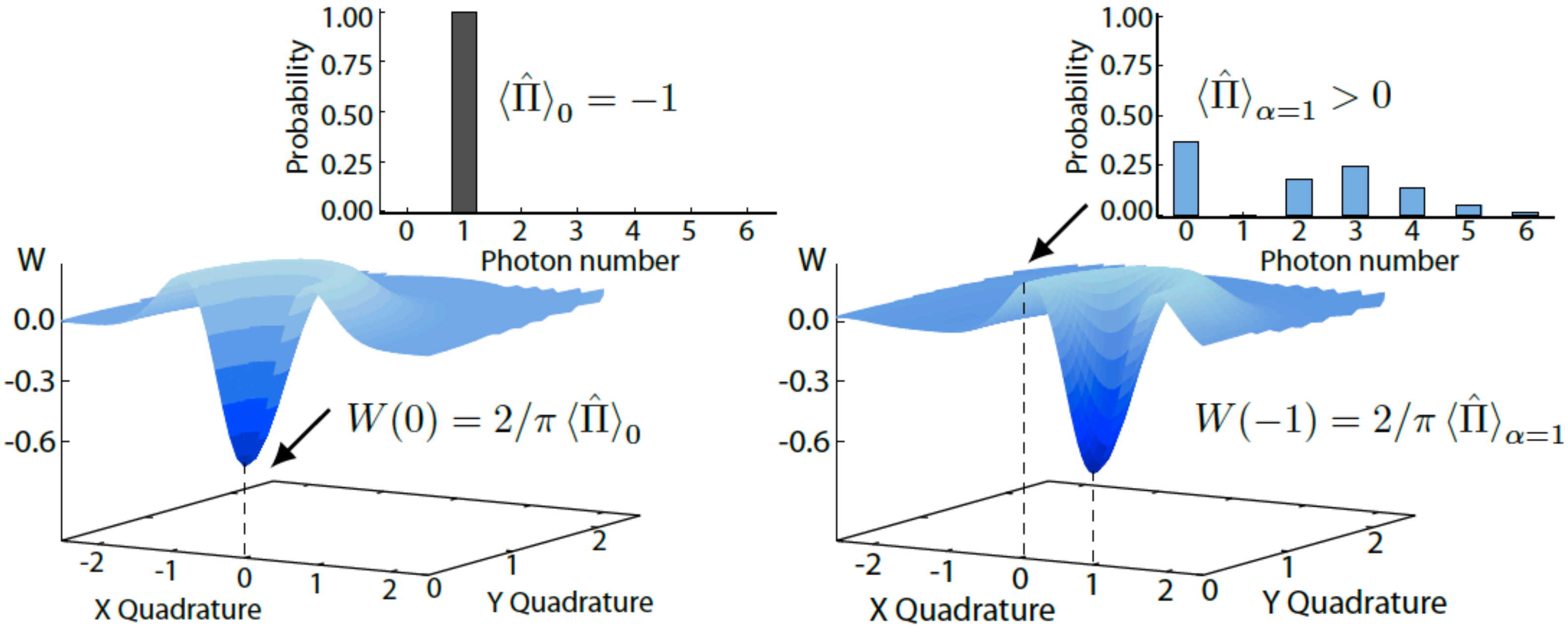
S. Wallentowitz, W. Vogel, PRA 53 (4528), 1996

K. Banaszek, K. Wodkiewicz, PRL 76 (4344), 1996

Direct probing of Wigner functions

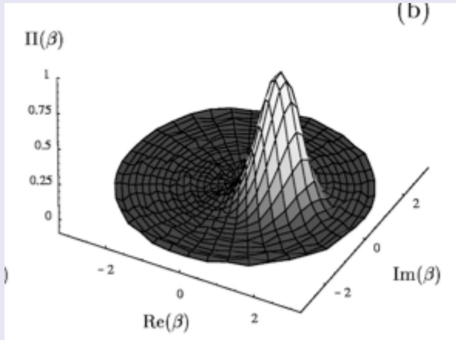
- $W(\alpha) = 2/\pi \langle\hat{\Pi}\rangle_{-\alpha}$, where $\hat{\Pi} = (-1)^{\hat{a}^\dagger \hat{a}}$
- measure parity via displaced state statistics
  $\langle\hat{\Pi}\rangle_{-\alpha} = P_0 - P_1 + P_2 - P_3 \ldots$
- ideal for studying non-classicality

SINGLE-PHOTON FOCK STATE $|1\rangle$:



Gaussian states

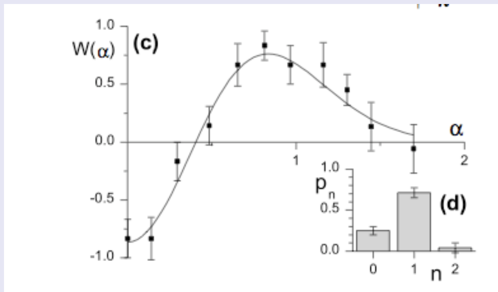
- Phase sensitive measurements



K. Banaszek, C. Radzewicz, K. Wodkiewicz, and J. S. Krasinski, PRA 60, 674 (1999)

Single photon in cavity

- Parity measured via atom-photon coupling

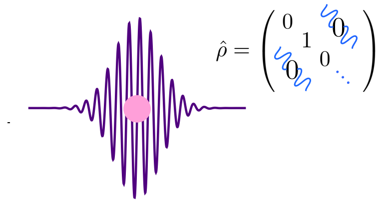


P. Bertet, A. Auffeves, P. Maioli, S. Osnaghi, T. Meunier, M. Brune, J. M. Raimond, and S. Haroche, PRL 89, 200402 (2002)

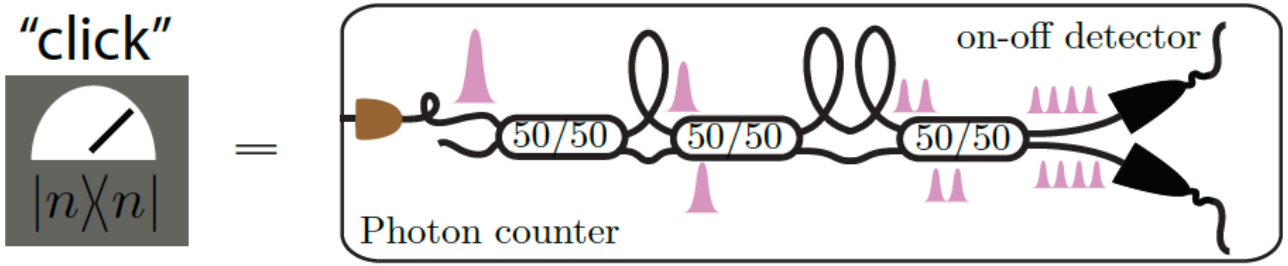
Our focus

Non-Gaussian travelling single-photon field

- Practical in quantum communications



Our implementation

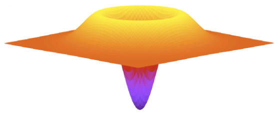


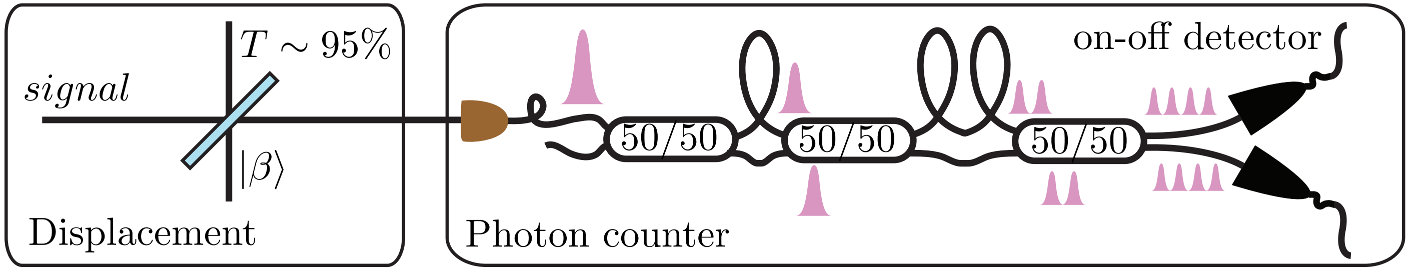
Loss-tolerant detection of photon statistics

- ✓ Single photon resolution via time-multiplexing
- ✓ Loss tolerant detection:
  efficient calibration of loss $\mathbf{L}(\eta)$ and convolution $\mathbf{C}$
- ✓ Statistics extracted by inverting $\vec{p}_{click} = \mathbf{CL}(\eta)\vec{\rho}$

D. Achilles et al., Opt. Lett. 28, 2387(2003); M. J. Fitch et al., PRA 68, 043814(2003)
D. Achilles et al., PRL 97, 043602(2005); M. Avenhaus et al., PRL 101, 053601(2008)

Our implementation





Loss-tolerant detection of photon statistics

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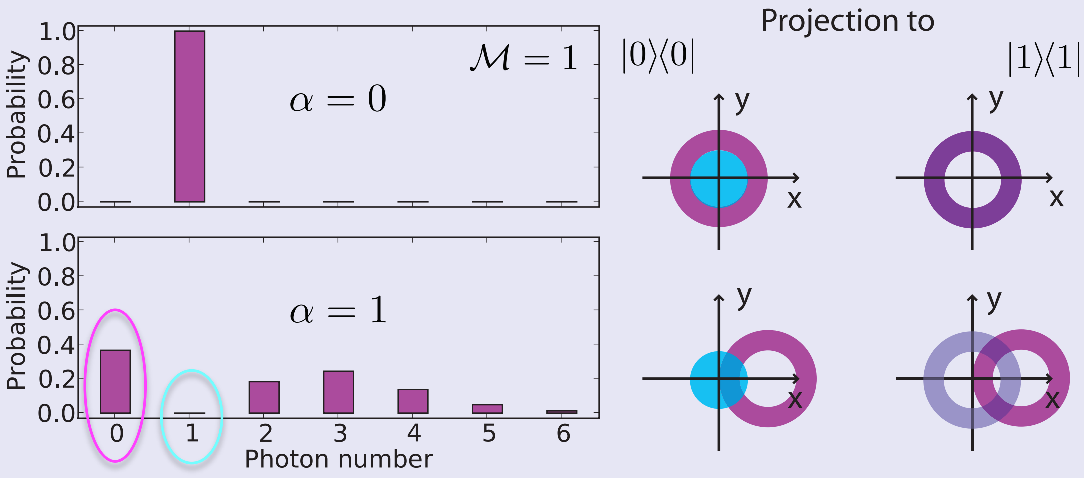
D. Achilles et al., PRL 97, 043602(2005); M. Avenhaus et al., PRL 101, 053601(2008)

Displacement – no phase sensitivity

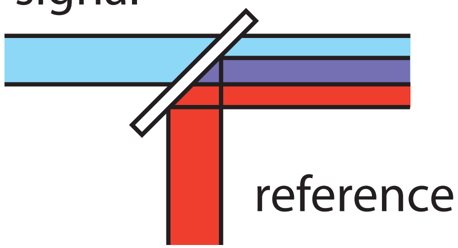
- ✓ Calibration by $|\alpha| = \sqrt{\langle n \rangle_{reference} / \eta}$

Measuring photon statistics in phase space

Statistics and mode overlap



W. Schleich and J. A. Wheeler, Nature 326, 574(1987)

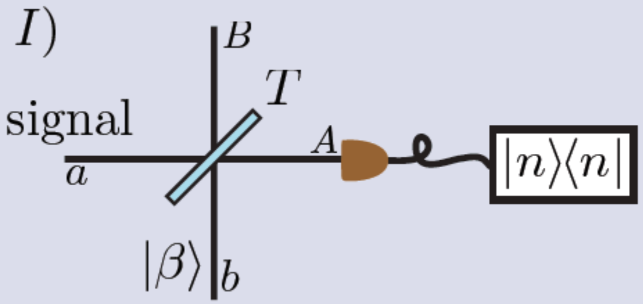


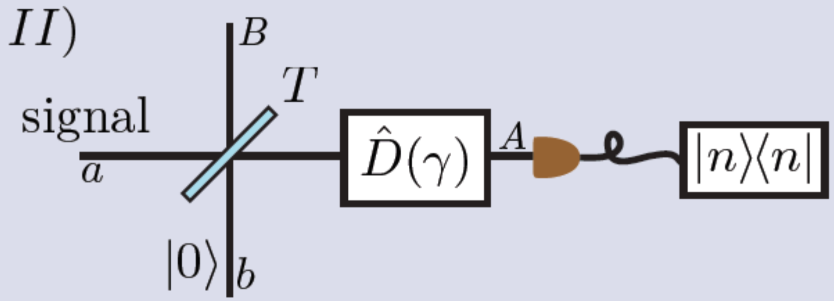
Spatial, spectral and temporal mismatch
→ introduces Poissonian background,
determined by mode overlap M

K. Banaszek et al., PRA 66, 043803(2002)
K. Laiho et al., NJP 11, 043012(2009)

Measuring photon statistics in phase space

Displacement

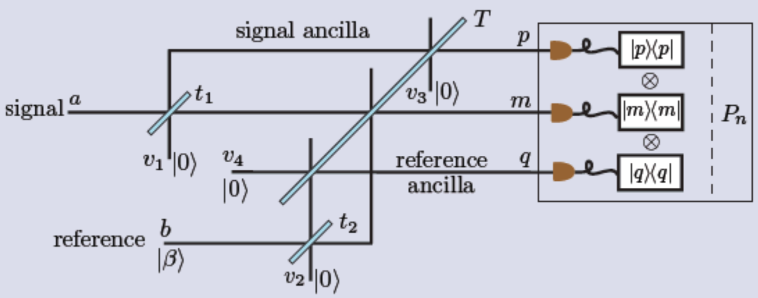




asymmetric BS = ideal displacement & loss

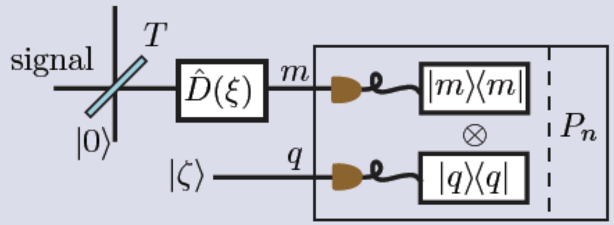
$$\vec{P}^{\hat{D}|\Psi\rangle} = L(\eta)\,\vec{\sigma}^{\hat{D}|\Psi\rangle}$$

Mode mismatch



=

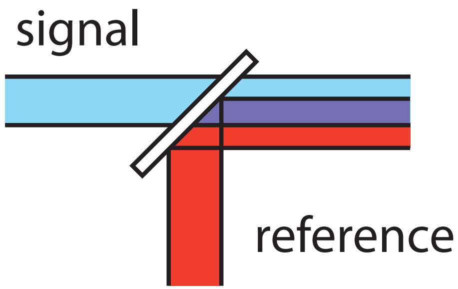
effective mode overlap parameter M



measured statistics = convolution of signal & one Poissonian statistcs

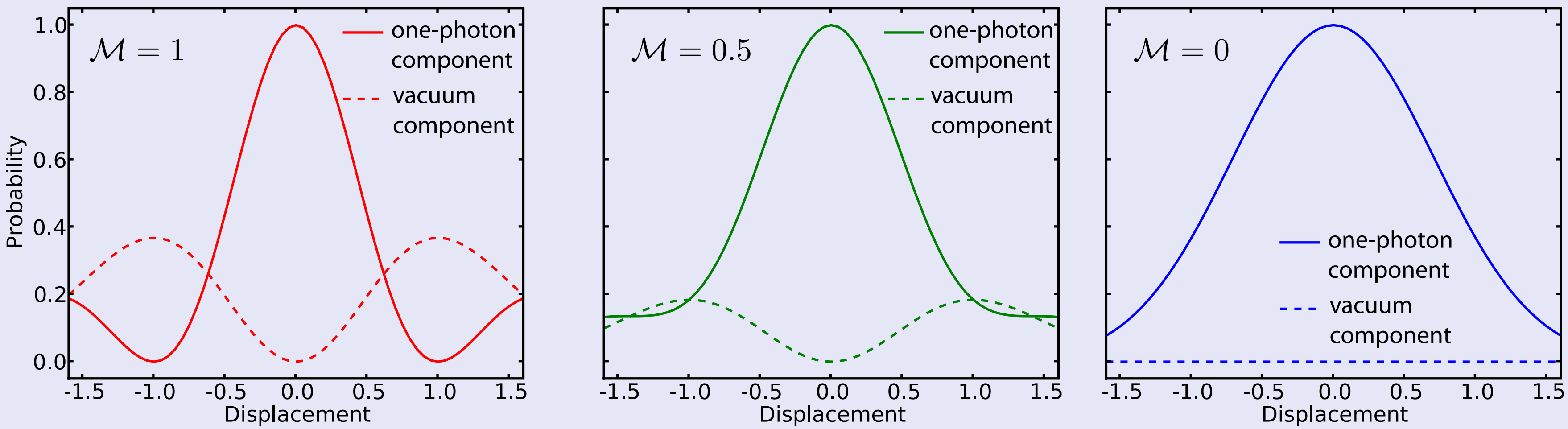
K. Laiho, M. Avenhaus, K.N. Cassemiro, Ch.S., NJP 11, 043012 (2009)

Mode overlap



Spatial, spectral and temporal mismatch
→ introduces Poissonian background,
determined by mode overlap M

p(0) and p(1) in dependence of displacement



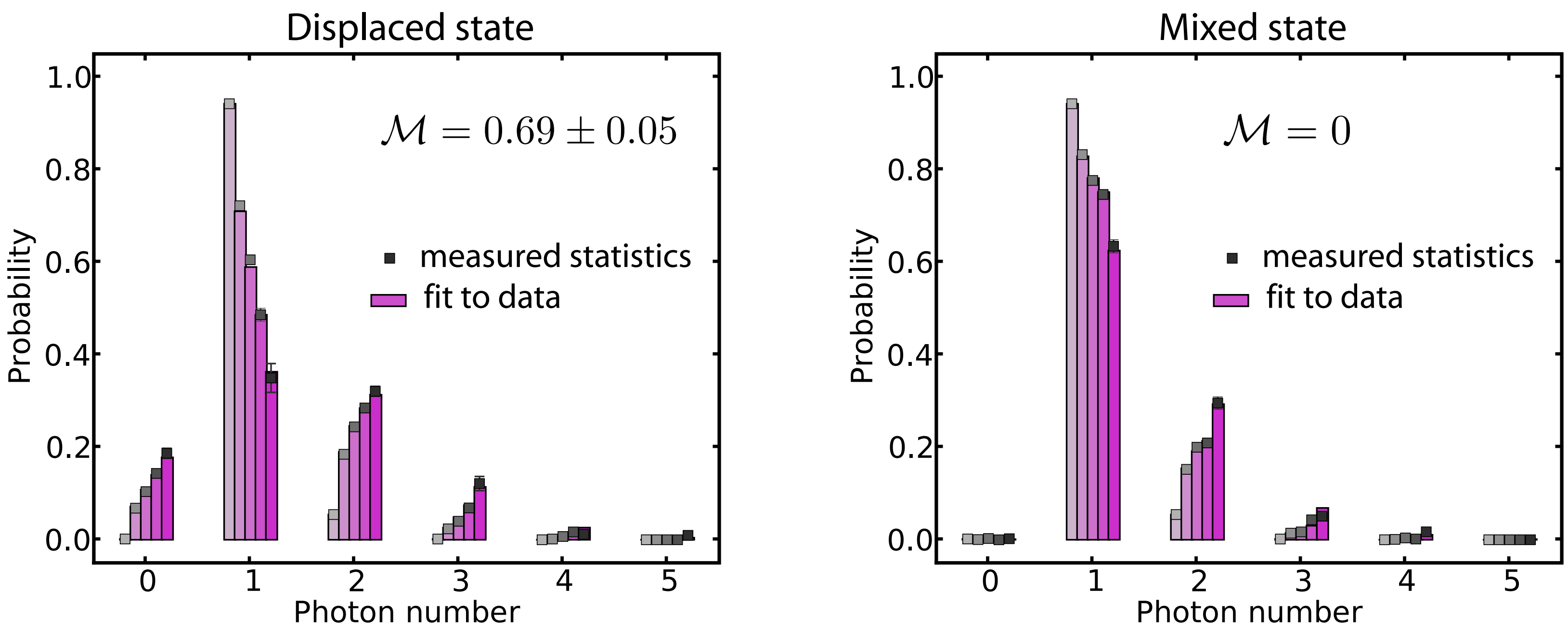
K. Laiho, M. Avenhaus, K.N. Cassemiro, Ch.S., NJP 11, 043012 (2009)

Displaced photon statistics

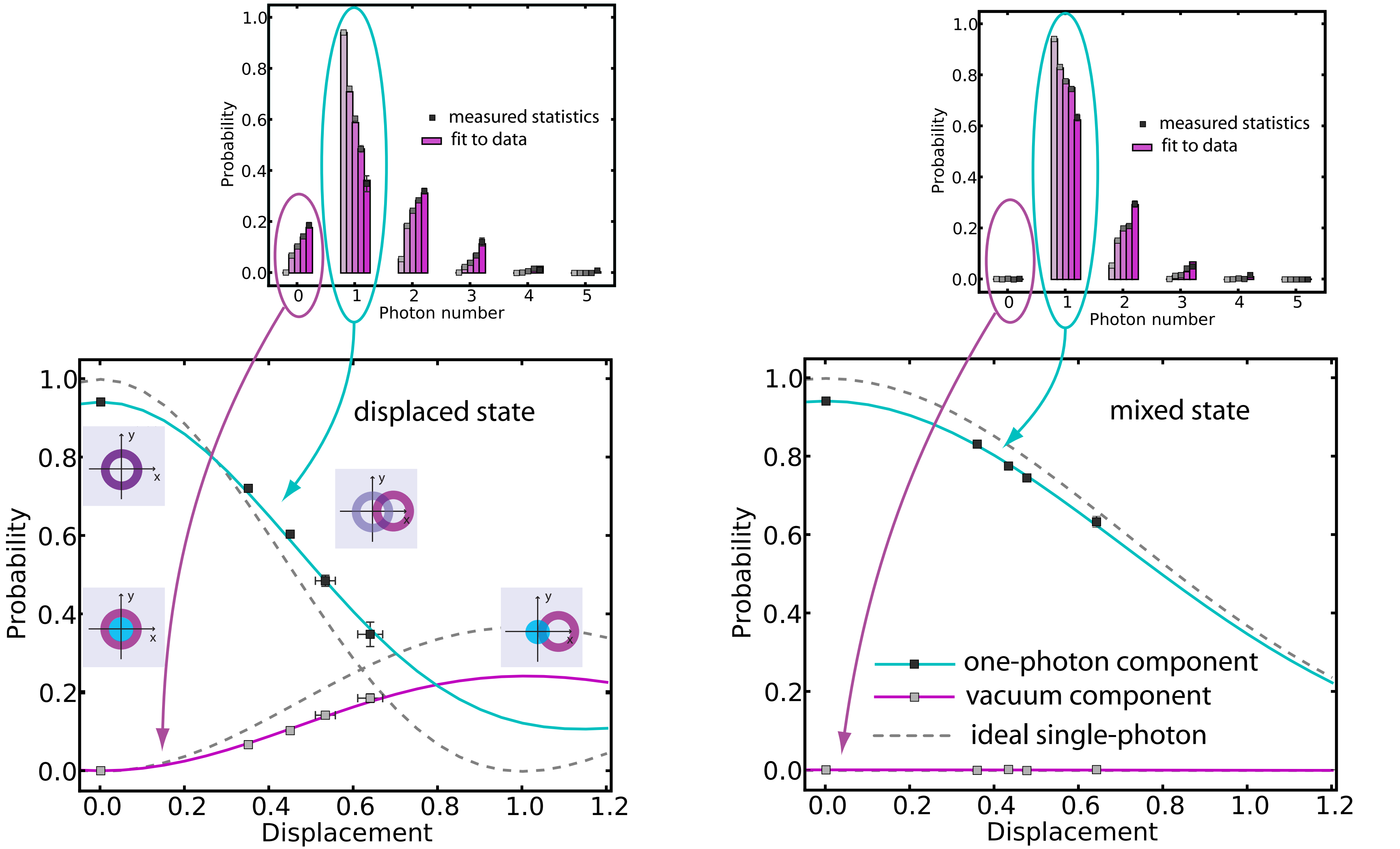
calibrated efficiency: $\eta = 0.165$

trigger events collected: $10^6 - 10^7$

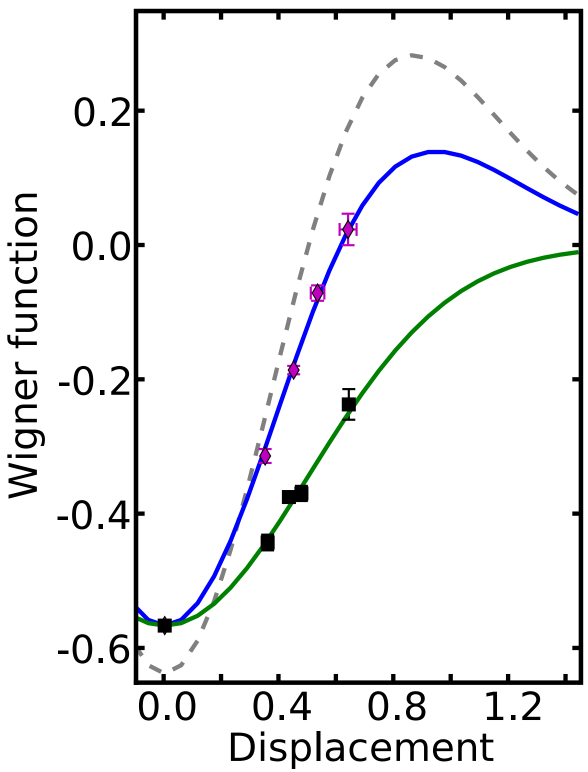
measurement time: $\sim 10^3 s$

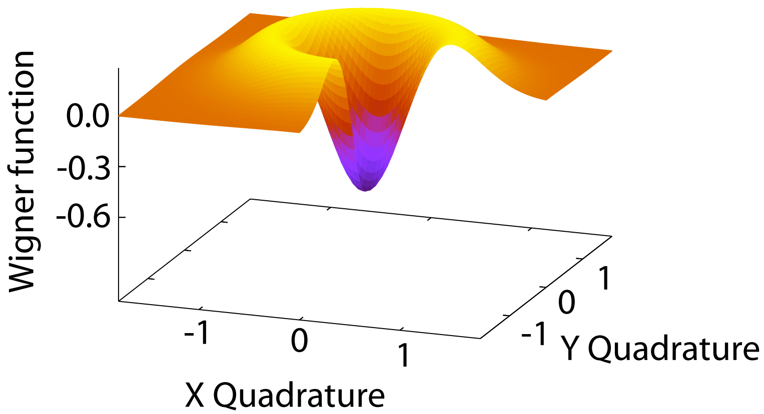


Displaced photon statistics



Probing of the Wigner function





Maximum negativity: -0.57

Overlap: 0.69 ± 0.05

K. Laiho, K.N. Cassemiro, D. Gross, Ch. S., Phys. Rev. Lett. 105, 253603 (2010)